Return–Risk Ratios Under Taxation

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The majority of theoretical asset allocation studies have been framed within a tax-free context. With the growth in both taxable and tax-deferred assets in recent years, it is important to review how the return–risk ratio can be adapted to the special needs of taxed portfolios. It turns out that this path leads to a number of surprising findings.

The subject of taxation is intrinsically complicated. This article makes use of a simplistic model for the purpose of illustrating differences between hypothetical taxed portfolios and their tax-free counterparts. For many reasons, this analysis should be viewed as a theoretical "first step" towards shedding light on one narrow facet of taxable investment rather than as a comprehensive basis for practical advice. The generic examples in this article should definitely not be interpreted as being directly applicable to any specific investor situation, nor should they be viewed as arguing for a given allocation, recommending any particular level of risk tolerance, or providing any form of tax advice whatsoever.

The natural intuition is that taxes always detract from investment returns. This intuition is, of course, true, but it does not answer how an investor's return–risk balance might be altered by taxation. This article addresses this issue by drawing upon a simple two-asset model composed of cash and a single equity asset, with the latter subject to an advantageous capital gains tax rate. It turns out that even though the taxed investor receives lower net returns, a differential tax structure can lead to return–risk ratios that are actually greater than the tax-free counterparts.

To achieve any return beyond the cash rate, an investor must be willing to accept some degree of risk. Indeed, any move into equities is motivated by some search for higher "returns." But the nature of the sought returns is often left surprisingly vague. The standard approach is to interpret this objective in terms of "expected returns." Return–risk ratios are almost always defined with expected returns as the numerator.

Expected returns, however, only have a 50% or so probability of being achieved (depending on the nature of the probability distribution). Another more-refined goal for risk taking might be to obtain some positive spread over the cash rate, and to do so with an acceptable probability. The standard return–risk ratios together with the equity percentage are the key determinants of such *spread probabilities.*

Most U.S. portfolios, from both individual (taxed) portfolios to even highly diversified institutional (tax-free) funds, have very nearly the same level of equity risk, as measured by a beta exposure of around 0.60. This equity percentage is often interpreted as representing the *maximum* tolerable volatility risk. Within the framework of this article, however, this common 0.60 beta may also approximate the *minimum* risk required to achieve acceptable spread probabilities.

With the level of risk being largely fixed in most portfolios, the return–risk ratio becomes the differentiating factor for achieving return targets, defined as spreads over the cash rate. And, even though taxes detract from overall returns, our analysis shows how, under some circumstances, taxed investors may actually enjoy relatively advantageous return–risk ratios. But over short-term horizons, given standard assumptions for equity returns and our two-asset model, the taxed investor's return–risk advantages are not sufficient to provide high probabilities for achieving positive spreads over the risk-free rate. To capture such excess returns with an acceptable probability, investors—be they taxed or tax-free—must move to significant equity positions, plan on five-year or longer horizons, or assume the presence of higher-than-standard equity return premiums.

As repeatedly noted, this article has been developed using a two-asset model that relies upon numerous simplifications and approximations. An entire universe of other asset classes and strategies were excluded from consideration. Moreover, the fundamental risk premium model is recognized as being only a crude representation of the many complexities involved in market behavior. The general thrust of our findings, however, points to the importance of considering other approaches for augmenting expected returns, such as active management or alpha-generating diversifications. The challenge is to find such sources where the resulting incremental returns have a reasonable prospect of being reliably positive over the long term, while adding only moderately to total fund volatility.

MINIMUM AND MAXIMUM RISK LEVELS

Many different formal approaches are available for finding the best balance of risk and return for a given investment situation. Some of the more sophisticated optimization techniques trace out *utility contours* of equally acceptable return–risk trade-offs. The actual practice of asset allocation, however, may have more fundamental roots in underlying behavioral patterns that are far removed from these more sophisticated approaches.

Recent studies by Leibowitz [2004] and Leibowitz and Bova [2005, 2007] showed that U.S. institutional portfolios, such as pension funds, foundations, and endowments, generally exhibit a common projected volatility (10%–11%) and the same high correlation with U.S. equities (90% or greater). Virtually all such funds appear to share the same embedded *beta sensitivity* of about 0.60 to U.S. equity that characterizes traditional allocations of 60% equity/ 40% fixed income. In normal times, this 0.60 beta sensitivity appears to apply even to funds that are broadly diversified across multiple asset classes and have only a minimal explicit percentage in U.S. equities. (Ironically, in periods of exceptional market turbulence, it may be these multi-asset diversified portfolios that will be more subject to correlation tightening, *stress-betas*, and consequently greater vulnerability than the traditional 60/40 funds (Leibowitz and Bova [2009]).

These same risk characteristics apply to many individual and private family portfolios as well (Leibowitz and Hammond [2004]), in spite of the wide divergence in investment goals, time horizons, spending needs, potential contingency events, back-up resources, and so forth. Several explanations have been proposed for this commonality in equity beta risk. They range from sophisticated optimization analyses to the simple observation that asset owners generally seek to minimize the probability of portfolio deterioration beyond some widely held threshold. The latter argument provides a basis for a maximum level of risk, but it fails to explain why almost all investors seem to move right up to this maximum limit. One answer might be that investors seek to maximize their expected return subject to this common risk limit.

The analysis developed in this article, however, presents another potential explanation. Investors must accept a certain degree of risk to achieve any significant probability of achieving a spread over the cash rate. And for a reasonable set of values, spread targets require the equity beta to be pushed up toward the 0.60 level seen in practice. Thus, it may be that roughly this same 0.60 beta risk actually serves, at least on an intuitive basis, both as a *minimum* for achieving reasonable return targets and as a *maximum* for risk tolerance.

THE TAXABLE FRONTIER

Our analysis will focus on two-asset portfolios consisting of varying mixtures of cash and equity. Exhibit 1 plots a return–risk "frontier" for a tax-free portfolio. The expected return of equities is assumed to be 7%, composed of a 4% rate for cash and a 3% risk premium. The pre-tax volatilities for cash and equity are assumed to be

EXHIBIT 1
A Tax-Free Return–Volatility Frontier

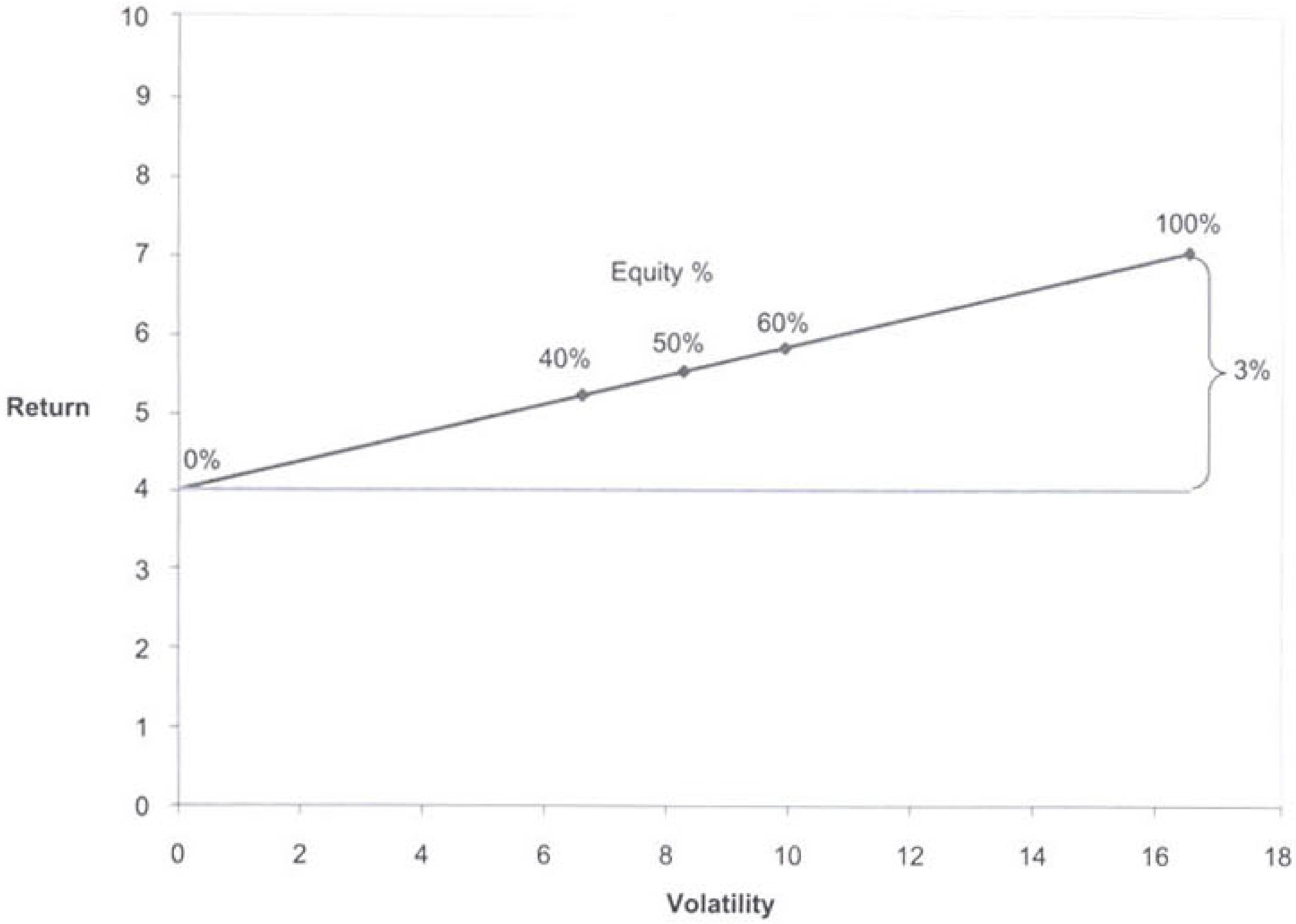


EXHIBIT 2
After-Tax Risk Premiums and Volatilities

	Tax Free	After Tax
Equity Return	7.00	5.60
Net Cash Return	4.00	2.40
Net Equity Premium over Cash	3.00	3.20
Equity Volatility	16.50	16.50
Tax Offset	0%	20%
Net Equity Volatility	16.50	13.20

Source: Morgan Stanley Research.

0% and 16.5%, respectively. (All return and volatility values were chosen solely for illustrative purposes.)

A large body of financial literature exists on the subject of risk premiums, virtually all referring to the institutional tax-free case. The estimates of the premium vary widely, with most studies focusing on a range of 3% to 7%. The preceding example was intentionally conservative in using the 3% figure.

Moving now to the taxable investor, suppose that all interest from cash is taxed at 40% and that all equity returns take the form of long-term capital gains (LTCG) taxed at 20%. (The assumption of a 20% tax on all sources of equity return is admittedly quite crude in that it neglects dividend income, short-term capital gains, and so forth.) These tax rates of 40% and 20% can be translated into "pull-through" rates of 60% and 80%, respectively, and represent the percentage of the pre-tax return that remains after taxes are applied. Exhibit 2 shows how taxes reduce the cash yield from 4% to 2.4%, and the equity return from 7% to 5.6%.

The net equity return over cash—or *after-tax equity risk premium*—is 3.2% (i.e., somewhat greater than the 3.0% tax-free premium). This at-first curious result is derived from the differential taxation of the two assets; the cash asset is impacted by the full 40% tax, while the lighter 20% capital gains tax covers the risk premium and also "shields" equity's underlying cash base (Arnott et al [2000], Leibowitz [2003]).

In a tax-free (TF) environment, any loss is fully experienced by the investor. But in an after-tax (AT) situation, the impact of a new loss depends on the overall gain or loss status of the investor. Needless to say, in practice, this situation can become quite complex. For this hypothetical example, the assumption is that the investor has an ample preexisting reserve of positive LTCGs with the associated tax liability immediately due at year-end. All new capital gains are assumed to be taxed at the same 20% rate.

Any new capital losses are treated as offsetting the prior LTCG liability. Consequently, 20% of any such loss is recaptured in the form of a reduction in the preexisting tax liability. Under these assumptions, the tax authorities are essentially sharing in both an investor's gains as well as losses. Just as any pre-tax gains are reduced by an 80% after-tax pull-through, so too will any losses be reduced by this same 80% factor. The bottom half of Exhibit 2 shows how this loss-offset reduces the effective volatility of 100% equity from the tax-free 16.50% to an after-tax 13.20%.

Exhibit 3 shows expected returns for the same equity percentage for the TF and AT cases. These values are then restructured in Exhibit 4 so that both TF and AT expected

EXHIBIT 3
Expected Returns for Various Equity Percentages

Tax Free			After Tax		
% Equity	Volatility	Expected Return	% Equity	Net Volatility	Net Expected Return
0	0	4	0	0.0	2.4
20	3.3	4.6	20	2.6	3.0
40	6.6	5.2	40	5.3	3.7
60	9.9	5.8	60	7.9	4.3
80	13.2	6.4	80	10.6	5.0
100	16.5	7	100	13.2	5.6

Source: Morgan Stanley Research.

EXHIBIT 4
Expected Returns for Various Equivalent Net Volatilities

Tax Free			After Tax		
Volatility	% Equity	Expected Return	Net Volatility	% Equity	Net Expected Return
0	0	4	0	0	2.4
3.3	20	4.6	3.3	25	3.2
6.6	40	5.2	6.6	50	4.0
9.9	60	5.8	9.9	75	4.8
13.2	80	6.4	13.2	100	5.6
16.5	100	7	—	—	—

Source: Morgan Stanley Research.

returns are aligned with the same net volatility level. As expected, the taxed investors' returns are seen to be consistently lower than the TF values.

This result is confirmed in Exhibit 5 which shows that the entire AT frontier falls below the TF case. Several interesting points of comparison deserve mention. The AT frontier starts at a significantly lower cash yield of 2.40%, but has a higher slope than the TF frontier. This higher slope suggests the possibility of a somewhat greater incremental return for a given movement toward additional equity exposure.

RETURN–RISK RATIOS

When the risk premium is divided by the net volatility, the resulting return–risk ratio is often referred to as the Sharpe ratio (SR). This ratio is often taken as one measure of the return incentive for moving toward riskier allocations. With a simple straight-line frontier, the SR corresponds to the frontier's slope. For a given volatility, higher risk premiums translate directly into higher SRs. To translate a tax-free SR into its taxable counterpart's SR*, both the return premium in the numerator and the volatility in the denominator must be restated in after-tax terms. As noted earlier, the AT frontier actually has a significantly greater slope than the TF line, and this relationship is reflected in the higher Sharpe ratios shown in Exhibit 6.

Exhibit 7 illustrates how different risk premiums lead to different one-year TF SRs and AT SR*s. By their construction, with the risk premium being defined as the difference between the expected return and the cash rate, the tax-free SRs depend only on the risk premium and the equity volatility. Hence, once an estimated value for the risk premium is set, the tax-free SR will have no further dependence on the cash rate. However, because of the benefit from differential taxation, the AT SR*s depend on both the risk premium and the cash rate. This effect is shown in Exhibit 7. After-tax SR*s are consistently higher than pre-tax SRs, and this advantage increases with higher cash rates. The formulas developed in the appendix show how the combination of the tax shield and the lower net volatility leads to the higher AT return–risk ratios.

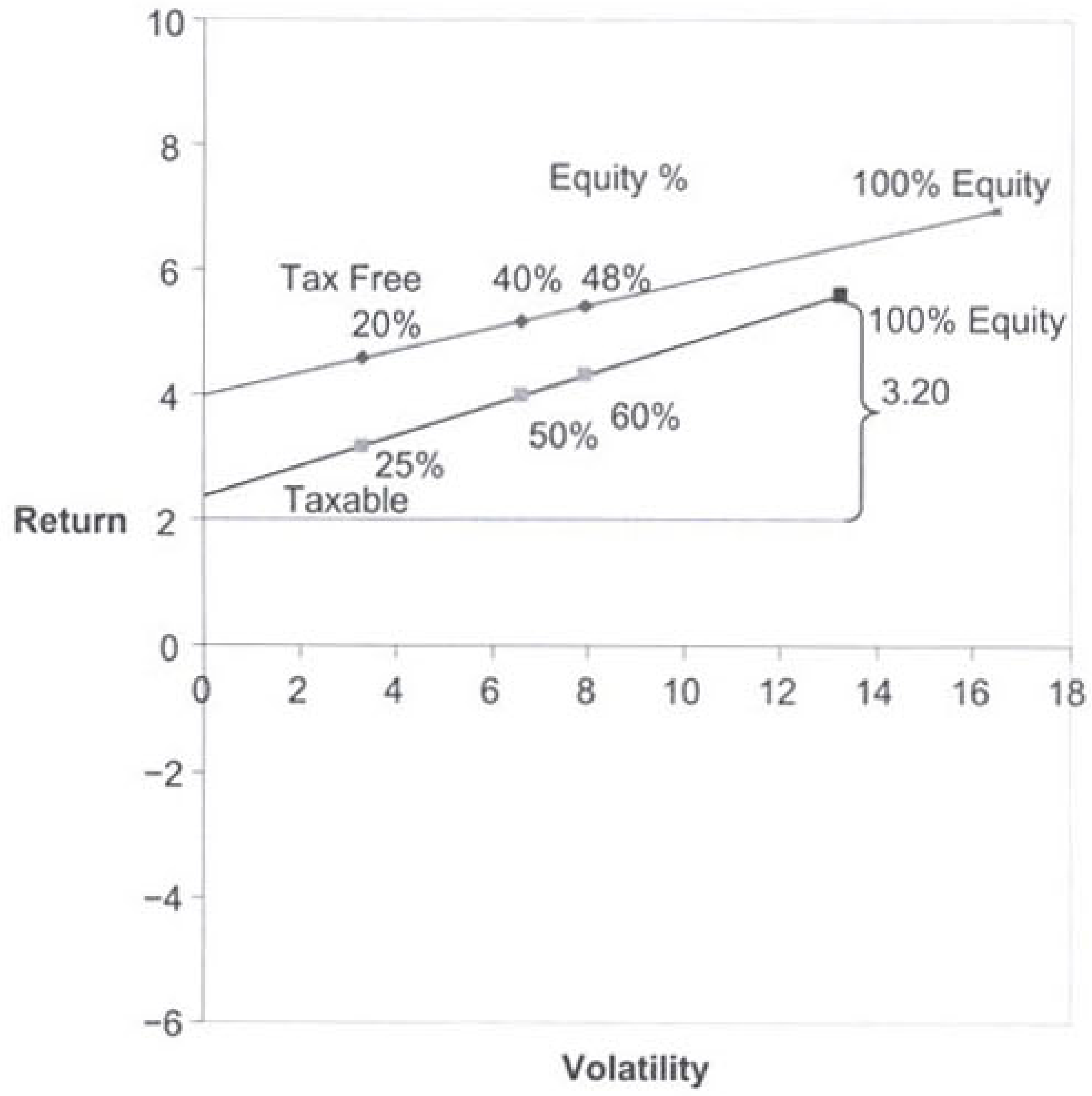
One problem with this approach is that TF and AT investors have different capacities for tolerating risk. Indeed, one could argue that, compared with tax-free institutions, taxed individuals generally do have a lower risk capacity because of more-limited time horizons, more-severe liquidity concerns, greater contingency risks, fewer back-up resources, and so forth.

RETURN TARGET AND SPREAD PROBABILITIES

These return–risk ratios also provide insight into the probability of achieving certain return targets. Exhibit 8 shows the probability of achieving various spreads over the 4% cash rate. The risk-free rate itself can always be obtained with 100% probability by simply investing totally in cash. For the base-case SR of 0.18, once any equity exposure is taken, such as 20% or 60%, the probability of besting 4% drops to 57%. In fact, it can be shown that a given SR is always associated with some

EXHIBIT 5

Simple Taxable Return–Volatility Diagram with 40% Interest Tax and 20% Capital Gains Tax



Source: Morgan Stanley Research.

EXHIBIT 6

Sharpe Return–Risk Ratios

	Tax Free	After Tax
Cash Rate	4.0	2.4
Risk Premium	3.0	3.2
Volatility	16.5	13.2
Sharpe Ratio	0.18	0.24

Source: Morgan Stanley Research.

EXHIBIT 7

After-Tax Risk Premims and Sharpe Ratios

			After-Tax Sharpe Ratios		
Pre-Tax	After-Tax	Pre-Tax	Pre-Tax Cash Rate		
Risk Premium	Risk Premium	Sharpe Ratio	2.0	4.0	6.0
3.0	2.4	0.18	0.21	0.24	0.27
5.0	4.0	0.30	0.33	0.36	0.39
7.0	5.6	0.42	0.45	0.48	0.52

Source: Morgan Stanley Research.

fixed probability of exceeding cash, regardless of the size of the equity investment (i.e., as long as it is beyond 0%). In other words, for a given SR value, there is a curious effect: as one departs from cash, any equity percentage—no matter how large or small—leads to a return–risk balance that reduces the probability of exceeding cash from 100% to some lower probability. Thus, an SR of 0.18 provides the same 57% probability of exceeding cash, whatever the cash rate or the equity percentage.

Any investment in equities obviously has a more ambitious goal of garnering returns in excess of the cash rate. An investor moves up the frontier to higher equity percentages in order to obtain higher expected returns. For symmetric normal probability distributions, expected returns represent only a 50% probability of being achieved. By themselves, expected returns indicate little about the probability of achieving a positive spread over the cash rate.

Exhibit 8 also shows the probability of achieving a 1% and 2% net spread over the cash rate. With a 0% equity exposure, no chance exists of obtaining any positive spread (i.e., the spread probability drops to 0%). To have any prospect of obtaining even a 1% excess return, an investor must accept some equity exposure. For the base-case SR of 0.18, Exhibit 8 shows spread probabilities of 45% and 53% for equity percentages of 20% and 60%, respectively. Even with the more aggressive 7% risk premium and the associated 0.42 SR, a 60% exposure is needed to bring the spread probability up to 63%. For the goal of a 2% spread, even this 60% equity exposure with the 0.42 SR only provides a 59% spread probability.

Although the preceding discussion focused on the probability of achieving various positive results, recognition of the risks entailed is also necessary. At the outset of the discussion, we noted that there is a 57% probability of exceeding the cash rate for both 20% and 60% equity positions. In fact, the 57% probability applies to any equity percentage. This result also implies that there is a 43% risk of falling below the cash rate—again, for any equity percentage! The risk of falling below the cash rate improves with longer time horizons, but at a surprisingly slow pace (Leibowitz and Krasker [1988]). The avoidance of negative returns represents a more stringent

EXHIBIT 8
One-Year Tax-Free Probability of Achieving Spread over Cash Rate

		Spread over Cash	0	0	0	1	1	1	2
Pre-Tax	Tax-Free	% Equity	0%	20%	60%	0%	20%	60%	60%
Risk Premium	Sharpe Ratio	Volatility	0	3.3	9.9	0	3.3	9.9	9.9
3	0.18		100	57	57	0	45	53	49
5	0.30		100	62	62	0	50	58	54
7	0.42		100	66	66	0	55	63	59

Source: Morgan Stanley Research.

risk criterion; the one-year probability of falling below 0% definitely depends on the equity percentage, with probabilities of 8% and 28% for equity percentages of 20% and 60%, respectively.

Returning to the goal of achieving positive spreads over cash, the depressingly low probabilities in Exhibit 8 apply to a one-year horizon. It could be argued, however, that any risky investment requires longer horizons to provide reasonable results. Using a common approximation, the square root of the number of years, the SR analysis can be extended to longer horizons. For example over five years, the TF equity volatility declines to an annualized $16.5/\sqrt{5} = 7.38$ and the one-year SR of 0.18 rises to $0.18(/\sqrt{5}) = 0.40$. (This simple approximation neglects a number of positive effects, such as compounding and reversion to the mean, as well as negative effects, such as volatility "drag.") In Exhibit 9, the TF spread probabilities are shown for a 60% equity position over both the one-year period and the five-year horizon. The spread probabilities are much improved for the five-year horizon, with the increase particularly notable for the higher SRs. For example, with a 7% risk premium, the probability of a 1% spread rises from 63% for one year to 77% over five years.

In Exhibit 10, the corresponding five-year TF and AT SRs are aligned for the case of a fixed 60% equity percentage. The higher AT SRs lead to only modest increases in the spread probabilities. But because of the loss offset in the AT case, the constant 60% equity position corresponds to a higher net volatility for the TF case than for the AT case. In Exhibit 11, the AT equity exposure is set at 60%, but the TF equity percentage is adjusted to equalize the net volatility (e.g., the AT 60% provides a net volatility of 3.54%, which equates to a 48% TF percentage). On this matched net-volatility basis, the AT spread probabilities tend to be well above the TF levels.

The natural intuition that taxes detract from investment returns is generally correct with respect to total returns. However, as shown in Exhibit 11, the incremental risk premiums, associated volatilities, and return–risk ratios may actually improve under certain tax environments. It is within this context that taxed investors might be said to have a somewhat greater incentive to accept equity risk.

EXHIBIT 9
One-Year and Five-Year Tax-Free Spread Probabilities for 60% Equity

	Spread over Cash	0	0	1	1	2	2
Pre-Tax	Horizon (Years)	1	5	1	5	1	5
Risk Premium	Annualized Volatility	16.50	7.38	16.50	7.38	16.50	7.38
3		57	66	53	57	49	48
5		62	75	58	67	54	59
7		66	83	63	77	59	69

Source: Morgan Stanley Research.

EXHIBIT 10

Five-Year After-Tax and Tax-Free Spread Probabilities for 60% Equity

	Spread over Cash	0	0	1	1	2	2
		TF	AT	TF	AT	TF	AT
Pre-Tax	Equity %	60%	60%	60%	60%	60%	60%
Risk Premium	Net Volatility	4.43	3.54	4.43	3.54	4.43	3.54
3		66	70	57	60	48	49
5		75	79	67	70	59	59
7		83	86	77	79	69	69

Source: Morgan Stanley Research.

EXHIBIT 11

Five-Year After-Tax and Tax-Free Spread Probabilities for 60% After-Tax Equity and 48% (Matched Volatility) Tax-Free Equity

	Spread over Cash	0	0	1	1	2	2
		TF	AT	TF	AT	TF	AT
Pre-Tax	Equity %	48%	60%	48%	60%	48%	60%
Risk Premium	Net Volatility	3.54	3.54	3.54	3.54	3.54	3.54
3		66	70	55	60	44	49
5		75	79	65	70	54	59
7		83	86	75	79	65	69

Source: Morgan Stanley Research.

GAIN AND LOSS OFFSET COMBINATIONS

The preceding examples focused on hypothetical taxes of 40% for cash interest and 20% for all forms of equity returns. Clearly, equity returns can take many different forms (e.g., dividends, short-term capital gains, realized long-term capital gains, returns of capital, long-term holdings to be donated, and so forth). As a further complication, carryforward losses can also play a role in reducing the tax liability from new capital gains.

Moreover, the realizations of gains can be deferred to longer-term horizons, which have the effect of further lowering the effective tax rate (barring any increase in overall tax rate over the deferral period!). Thus, the effective tax rates on different mixtures of these return components can range from the highest income tax rate to an arguably negligible rate for holdings that are properly contributed (or stepped up!).

To this point, the tax rate on equities has been assumed to apply both to new expected returns as well as to the offsets from losses. On the one hand, capital losses may be least damaging when they can act as offsets to realized high-tax liabilities, such as short-term capital gains. On the other hand, such losses may magnify the pain when they erode positions that would otherwise be subject to low tax rates. And when net losses can be carried forward, they can act as offsets against gains in future years. These considerations highlight the need for a multi-period analysis that goes well beyond the confines of this article's simple "cash out" model.

The formulas developed in the appendix can be used to estimate the equity percentages and net returns associated with other tax rates for interest income, capital gains, and loss offsets.

CONCLUSIONS

Taxes have a material impact on prospective returns, and these effects should ideally be incorporated into the allocation process. The typical tax situation will reduce the expected level of returns, but the net equity premium and the return–risk ratio may actually both increase. Thus, within this simple framework, the incentive for risk taking may theoretically be somewhat greater for taxed investors.

For many reasons, however, taxed investors may have less ability to accept risk than tax-free institutions. Moreover, to obtain significant probabilities of achieving even modest excess returns, both tax-free and taxed investors must move toward higher equity positions, have long-term horizons, and embrace relatively aggressive estimates regarding equity premiums.

The simple two-asset model we have presented in this article has excluded any consideration of incremental returns from active management or from diversification into alpha-generating asset classes. Such incremental returns would clearly improve the prospect of achieving a specified return target, especially if they could be reliably achieved without material increases in the total fund volatility.

APPENDIX

The basic model treats three simple forms of taxation: 1) income taxed at rate $(1 - P_I)$; 2) equity that produces only capital gains taxed at the advantageous rate $(1 - P_G)$; and 3) capital losses that provide offsets to tax liabilities that would have been taxed at rate $(1 - P_L)$. The investment model assumes a pre-tax expected return of y for fixed income and $(y + r)$ for equity, where r denotes the equity risk premium. The equity volatility risk is denoted by s, and the fixed-income asset is treated as risk free.

A tax-free account with an equity percentage w would have the expected return,

$$
\begin{aligned}
R(w) &= w(y+r)+(1-w)y \\
&= wr + y \\
R(w) &= w\left(\frac{r}{\sigma}\right)\sigma + y \\
&= w(SR)\sigma + y
\end{aligned}
$$

where SR is the TF Sharpe ratio,

$$SR = \left(\frac{r}{\sigma}\right)$$

A taxable account with an equity percentage w would have net return, $R^*(w)$

$$
\begin{aligned}
R^*(w) &= wP_G(r+y)+(1-w)P_I y \\
&= w\left[P_G r + y(P_G - P_I)\right] + P_I y \\
&= wr^* + P_I y
\end{aligned}
$$

where r^* is now the taxable risk premium for a 100% equity portfolio.

$$r^* = P_G r + y(P_G - P_I)$$

The 100% equity portfolio would have a net volatility of $P_L\sigma$, so the loss-offset factor reduces the net volatility to $P_L\sigma$ and the taxable Sharpe ratio SR* becomes

$$SR^* = \frac{r^*}{P_L} = \frac{P_G}{P_L}SR + y\left(\frac{P_G - P_I}{P_L}\right)$$

In the special case when $P_L = P_G$,

$$
\begin{aligned}
SR^* &= \frac{r}{\sigma} + \frac{y}{\sigma}\left(1 - \frac{P_I}{P_L}\right) \\
&= SR + \frac{y}{\sigma}\left(1 - \frac{P_I}{P_L}\right)
\end{aligned}
$$

The taxable portfolio expected return can then be written as

$$
\begin{aligned}
R^*(w) &= w\left[P_G r + y(P_G - P_I)\right] + P_I y \\
&= wr^* + P_I y \\
&= w\left(\frac{r^*}{P_L\sigma}\right)P_L\sigma + P_I y \\
&= w(SR^*)P_L\sigma + P_I y
\end{aligned}
$$

For a normal distribution, a taxed portfolio with equity percentage w would have a pth percentile return,

$$R^*(w) - k_p(wP_L\sigma)$$

where k_p is the pth percentile,

$$\text{Prob}\{z \geq R^*(w) - k_p(wP_L\sigma)\} = p$$

To have a probability p that the return exceeds the after-tax cash rate $(P_I y)$ by some spread s, then

$$R^*(w) - k_p(wP_L\sigma) = P_I y + s$$

or

$$w(SR^*)P_L\sigma + P_I y - k_p(wP_L\sigma) = P_I y + s$$

or

$$(w_L P_L \sigma)(SR^* - k_p) = s$$

This equality can be solved for the percentile,

$$k_p = SR^* - \left(\frac{s}{wP_L\sigma}\right) \quad w > 0$$

When $s = 0$, k_p depends only on SR^*, so the Sharpe ratio itself determines the probability of exceeding the after-tax risk-free rate. Note that the same probability applies to all the positive equity weights.

The preceding development was implicitly based on a one-year investment horizon. For a horizon of N years, a simple random walk would lead to roughly the same annualized return, but with an annualized volatility reduced by $1/\sqrt{N}$, so that

$$k_p = \sqrt{N}\left[SR^* - \left(\frac{s}{wP_L\sigma}\right)\right] \quad w > 0$$

For the base-case example,

$$\begin{aligned} r^* &= [P_G r + \gamma(P_G - P_I)] \\ &= 0.8(3) + (4)(0.8 - 0.6) \\ &= 3.2 \end{aligned}$$

and the one-year SR^* is

$$\begin{aligned} SR^* &= \frac{r^*}{P_L\sigma} \\ &= \frac{3.2}{(0.8)16.5} \\ &= 0.24 \end{aligned}$$

while over five years, for $s = 1$ and a 60% equity position

$$\begin{aligned} k_p &= \sqrt{5}\left[0.24 - \left(\frac{1}{(0.6)(0.8)(16.5)}\right)\right] \\ &= 2.24[0.24 - 0.13] \\ &= 0.25 \end{aligned}$$

for a probability of 60%.

ENDNOTE

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